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Geometric Phase in a Λ-type k-photon Jaynes-Cummings Model with Imaginary Photon Process

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Abstract By using the Lewis–Riesenfeld invariant theory, we have studied the dynamical and the geometric phases in a generalized time-dependent Λ-type k-photon Jaynes-Cummings model with imaginary photon process. We find that the geometric phases in a cycle case are independent of the frequency of the photon field, the coupling coefficient between photons and atoms, and the atom transition frequency. If we use the more accuracy device, the geometric phases in this process may be observed, and the geometric phases in this process have the observable physical effect.

Keywords Geometric phase · Generalized Jaynes-Cummings model

1 Introduction

It is known that the concept of geometric phase was first introduced by Pancharatnam [1] in studying the interference of classical light in distinct states of polarization. Berry [2] found the quantal counterpart of Pancharatnam's phase in the case of cyclic adiabatic evolution. The extension to non-adiabatic cyclic evolution was developed by Aharonov and Anandan [3]. Samuel et al. [4] generalized the pure state geometric phase by extending it to non-cyclic evolution and sequential projection measurements. The geometric phase is a consequence of quantum kinematics and is thus independent of the detailed nature of the dynamical origin of the path in state space. Mukunda and Simon [5] gave a kinematic approach

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by taking the path traversed in state space as the primary concept for the geometric phase. Further generalizations and refinements, by relaxing the conditions of adiabaticity, unitarity, and cyclicity of the evolution, have since been carried out [6]. Recently, the geometric phase of the mixed states has also been studied [7–9].

As we known that the quantum invariant theory proposed by Lewis and Riesenfeld [10] is a powerful tool for treating systems with time-dependent Hamiltonians. It was generalized by introducing the concept of basic invariants and used to study the geometric phases [11–14] in connection with the exact solutions of the corresponding time-dependent Schrödinger equations. The discovery of Berry's phase is not only a breakthrough in the older theory of quantum adiabatic approximations, but also provides us with new insights in many physical phenomena. The concept of Berry's phase has been developed in some different directions [15–28]. Based on the studies of the Jaynes-Cummings model [29–35], and by using the Lewis–Riesenfeld invariant theory, we shall study the geometric phase in a generalized time-dependent Λ-type k-photon Jaynes-Cummings model with imaginary photon process.

2 Model

The Hamiltonian of the generalized time-dependent Λ-type k-photon Jaynes-Cummings model with imaginary photon process for three-level atoms interacting with light can be written as

$$\begin{aligned}\hat{H} &= \omega(t)\hat{a}^\dagger\hat{a} + \omega_0(t)\hat{S}_z + g(t)(\hat{a}^k\hat{S}_+ + \hat{a}^{\dagger k}\hat{S}_-)\\ &\quad + g(t)(\hat{a}^{\dagger k}\hat{S}_+e^{2i\omega(t)t} + \hat{a}^k\hat{S}_-e^{-2i\omega(t)t}),\end{aligned} \tag{1}$$

where the last term $g(t)(\hat{a}^{\dagger k}\hat{S}_+e^{2i\omega(t)t} + \hat{a}^k\hat{S}_-e^{-2i\omega(t)t})$ denotes the imaginary photon process. $\hat{S}_\pm$ and $\hat{S}_z$ are

$$\hat{S}_+ = \frac{1}{\sqrt{2}}|3\rangle(\langle 1| + \langle 2|), \qquad \hat{S}_- = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)\langle 3|, \tag{2}$$

$$\hat{S}_z = \frac{1}{2}\left[|3\rangle\langle 3| - \frac{1}{2}(|1\rangle + |2\rangle)(\langle 1| + \langle 2|)\right], \tag{3}$$

and they satisfy the commutation relations $[\hat{S}_+, \hat{S}_-] = 2\hat{S}_z$ and $[\hat{S}_z, \hat{S}_\pm] = \pm\hat{S}_\pm$. This model describes a Λ-type photon transition of a three-level atom whose two lower levels are nearly degenerate. $\hat{a}(\hat{a}^\dagger)$ are the photon annihilation (creation) operators, $\omega(t)$ is the frequency of the photon field, $g(t)$ is the coupling coefficient between photons and atoms, and $\omega_0(t)$ is the atom transition frequency. $\hat{S}_+$, $\hat{S}_-$, and $\hat{S}_z$ are defined as

$$\begin{aligned}&\hat{S}_+ = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \qquad \hat{S}_- = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 1 & 0 & 0\end{pmatrix},\\ &\hat{S}_z = \frac{1}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & -\frac{1}{2} & \frac{-1}{2}\end{pmatrix}.\end{aligned} \tag{4}$$

If introducing the supersymmetric generators $\hat{P}_+ = \hat{a}^{\dagger k}\hat{S}_+$, and $\hat{P}_- = \hat{a}^k\hat{S}_-$, one has

$$\{\hat{P}_+, \hat{P}_-\} = \frac{1}{2}\begin{pmatrix} 2\hat{a}^{\dagger k}\hat{a}^k & 0 & 0 \\ 0 & \hat{a}^k\hat{a}^{\dagger k} & \hat{a}^k\hat{a}^{\dagger k} \\ 0 & \hat{a}^k\hat{a}^{\dagger k} & \hat{a}^k\hat{a}^{\dagger k} \end{pmatrix} \equiv \hat{\hat{M}}. \tag{5}$$

It is easy to find that $\hat{\hat{M}}$, $\hat{a}^\dagger\hat{a}$, $\hat{S}_\pm$, $\hat{S}_z$, and $\hat{P}_\pm$ are the supersymmetric generators and they form the supersymmetric Lie algebra, namely

$$\hat{P}_-^2 = \hat{P}_+^2 = 0, \qquad [\hat{P}_+, \hat{P}_-] = 2\hat{\hat{M}}\hat{S}_z, \qquad [\hat{\hat{M}}, \hat{P}_-] = [\hat{\hat{M}}, \hat{P}_+] = [\hat{\hat{M}}, \hat{S}_z] = 0, \tag{6}$$

$$[\hat{\hat{M}}, \hat{a}^\dagger\hat{a}] = 0, \qquad \{\hat{P}_-, \hat{S}_z\} = \{\hat{P}_+, \hat{S}_z\} = 0, \tag{7}$$

$$\hat{S}_z(\hat{P}_+ - \hat{P}_-) = \frac{1}{2}(\hat{P}_+ + \hat{P}_-), \qquad (\hat{P}_+ - \hat{P}_-)^2 = -\hat{\hat{M}}, \tag{8}$$

$$[\hat{P}_-, \hat{a}^\dagger\hat{a}] = k\hat{P}_-, \qquad [\hat{P}_+, \hat{a}^\dagger\hat{a}] = -k\hat{P}_+, \qquad [\hat{P}_-, \hat{S}_z] = \hat{P}_-, \qquad [\hat{P}_+, \hat{S}_z] = -\hat{P}_+, \tag{9}$$

where {, } stands for the anticommuting bracket.

It is known that the imaginary photons are absorbed by the atoms in a rapid speed. If we use the more accuracy device, the geometric phase in this process may be observed, which is the starting point of this paper. Based on this idea, we consider the following Hamiltonian of the system

$$\hat{H} = \omega(t)\hat{a}^\dagger\hat{a} + \omega_0(t)\hat{S}_z + g(t)(P_+e^{2i\omega(t)t} + P_-e^{-2i\omega(t)t}). \tag{10}$$

It is easy to find that

$$\hat{\hat{M}}\begin{pmatrix} |n\rangle \\ 0 \\ 0 \end{pmatrix} = \frac{n!}{(n-k)!}\begin{pmatrix} |n\rangle \\ 0 \\ 0 \end{pmatrix}, \tag{11}$$

$$\hat{\hat{M}}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ |n+1\rangle \\ |n+1\rangle \end{pmatrix} = \frac{(n+k+1)!}{(n+1)!}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ |n+1\rangle \\ |n+1\rangle \end{pmatrix}, \tag{12}$$

$$\hat{\hat{M}}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ |n+1\rangle \\ -|n+1\rangle \end{pmatrix} = 0, \tag{13}$$

so we can restrict our study in the sub-Hilbert space of the supersymmetric quasi-algebra $\hat{\hat{M}}$, $\hat{a}^\dagger\hat{a}$, $\hat{P}_\pm$, $\hat{S}_\pm$, and $\hat{S}_z$. Below, we replace operator $\hat{\hat{M}}$ with the particular eigenvalues $\frac{n!}{(n-k)!}$, $\frac{(n+k+1)!}{(n+1)!}$, and 0, respectively.

3 Dynamical and Geometric Phases

For self-consistent, we first illustrate the Lewis–Riesenfeld (L–R) invariant theory [10]. For a one-dimensional system whose Hamiltonian $\hat{H}(t)$ is time-dependent, then there exists an operator $\hat{I}(t)$ called invariant if it satisfies the equation

$$i\frac{\partial \hat{I}(t)}{\partial t} + [\hat{I}(t), \hat{H}(t)] = 0. \tag{14}$$

The eigenvalue equation of the time-dependent invariant $|\lambda_n, t\rangle$ is given

$$\hat{I}(t)|\lambda_n, t\rangle = \lambda_n|\lambda_n, t\rangle, \tag{15}$$

where $\frac{\partial \lambda_n}{\partial t} = 0$. The time-dependent Schrödinger equation for this system is

$$i\frac{\partial|\psi(t)\rangle_s}{\partial t} = \hat{H}(t)|\psi(t)\rangle_s. \tag{16}$$

According to the L–R invariant theory, the particular solution $|\lambda_n, t\rangle_s$ of (16) is different from the eigenfunction $|\lambda_n, t\rangle$ of $\hat{I}(t)$ only by a phase factor $\exp[i\delta_n(t)]$, i.e.,

$$|\lambda_n, t\rangle_s = \exp[i\delta_n(t)]|\lambda_n, t\rangle, \tag{17}$$

which shows that $|\lambda_n, t\rangle_s$ $(n = 1, 2, \ldots)$ forms a complete set of the solutions of (16). Then the general solution of the Schrödinger equation (16) can be written by

$$|\psi(t)\rangle_s = \sum_n C_n \exp[i\delta_n(t)]|\lambda_n, t\rangle, \tag{18}$$

where

$$\delta_n(t) = \int_0^t dt'\langle\lambda_n, t'|i\frac{\partial}{\partial t'} - \hat{H}(t')|\lambda_n, t'\rangle, \tag{19}$$

and $C_n = \langle\lambda_n, 0|\psi(0)\rangle_s$.

For the system described by Hamiltonian (10), we can define the following invariant

$$\hat{I}(t) = \alpha(t)\hat{P}_- + \alpha^*(t)\hat{P}_+ + \beta(t)\hat{S}_z. \tag{20}$$

Substituting (10) and (20) into (14), one has the auxiliary equations

$$i\dot{\alpha}(t) + [k\omega(t) + \omega_0(t)]\alpha(t) - g(t)\beta(t)e^{-2i\omega(t)t} = 0, \tag{21}$$

$$i\dot{\beta}(t) + 2g(t)\bar{M}[\alpha^*(t)e^{-2i\omega(t)t} - \alpha(t)e^{2i\omega(t)t}] = 0, \tag{22}$$

where dot denotes the time derivative, and $\bar{M}$ denotes the eigenvalue of operator $\hat{\bar{M}}$.

In order to obtain a time-independent invariant, we can introduce the unitary transformation operator $\hat{V}(t) = \exp[\xi(t)\hat{P}_- - \xi^*(t)\hat{P}_+]$. It is easy to find that when satisfying the following relations

$$\sin(2\sqrt{\bar{M}}|\xi(t)|) = \frac{\sqrt{\bar{M}}[\alpha(t)\xi^*(t) + \alpha^*(t)\xi(t)]}{|\xi(t)|}, \qquad \beta(t) = \cos(2\sqrt{\bar{M}}|\xi(t)|), \tag{23}$$

and

$$\alpha(t) - \frac{\beta(t)\xi(t)}{2\sqrt{\bar{M}}|\xi(t)|}\sin(2\sqrt{\bar{M}}|\xi(t)|) + \frac{\xi(t)[\alpha(t)\xi^*(t) + \alpha^*(t)\xi(t)]}{2|\xi(t)|^2}[\cos(2\sqrt{\bar{M}}|\xi(t)|) - 1] = 0, \tag{24}$$

then a time-independent invariant appears

$$\hat{I}_V \equiv \hat{V}^\dagger(t)\hat{I}(t)\hat{V}(t) = \hat{S}_z. \tag{25}$$

According to (23) and (24), we can select

$$\xi(t) = \frac{\theta(t)}{2\sqrt{\bar{M}}}\exp[i\gamma(t)], \qquad \alpha(t) = \frac{\sin\theta(t)}{2\sqrt{\bar{M}}}\exp[i\gamma(t)], \quad \theta(t) = 2\sqrt{\bar{M}}|\xi(t)|. \tag{26}$$

From (26), the invariant $\hat{I}(t)$ in (20) becomes

$$\hat{I}(t) = \frac{\sin\theta(t)}{2\sqrt{\bar{M}}}\{\exp[i\gamma(t)]\hat{P}_- + \exp[-i\gamma(t)]\hat{P}_+\} + \cos\theta(t)\hat{S}_z. \tag{27}$$

By using the Baker–Campbell–Hausdorff formula [35]

$$\hat{V}^\dagger(t)\frac{\partial\hat{V}(t)}{\partial t} = \frac{\partial\hat{L}}{\partial t} + \frac{1}{2!}\left[\frac{\partial\hat{L}}{\partial t},\hat{L}\right] + \frac{1}{3!}\left[\left[\frac{\partial\hat{L}}{\partial t},\hat{L}\right],\hat{L}\right] + \frac{1}{4!}\left[\left[\left[\frac{\partial\hat{L}}{\partial t},\hat{L}\right],\hat{L}\right],\hat{L}\right] + \cdots, \tag{28}$$

with $\hat{V}(t) = \exp[\hat{L}(t)]$, it is easy to find that when satisfying the following equation

$$\begin{aligned} &i\dot{\xi}(t) + i\frac{\xi(t)[\dot{\xi}(t)\xi^*(t) - \dot{\xi}^*(t)\xi(t)]}{4\sqrt{\bar{M}}|\xi(t)|^3}[\sin(2\sqrt{\bar{M}}|\xi(t)|) - 2\sqrt{\bar{M}}|\xi(t)|] \\ &\quad + \frac{[\omega_0(t) + k\omega(t)]\xi(t)}{2\sqrt{\bar{M}}|\xi(t)|}\sin(2\sqrt{\bar{M}}|\xi(t)|) - \frac{ge^{-2i\omega(t)t}}{2}[\cos(2\sqrt{\bar{M}}|\xi(t)|) - 1] \\ &\quad - ge^{-2i\omega(t)t} - \frac{g\xi^2(t)e^{2i\omega(t)t}}{2|\xi(t)|^2}[\cos(2\sqrt{\bar{M}}|\xi(t)|) - 1] = 0, \end{aligned} \tag{29}$$

one has

$$\begin{aligned} \hat{H}_V(t) &= \hat{V}^\dagger(t)\hat{H}(t)\hat{V}(t) - i\hat{V}^\dagger(t)\frac{\partial\hat{V}(t)}{\partial t} = \omega(t)\hat{a}^\dagger\hat{a} + \{\omega_0(t)\cos\theta(t) \\ &\quad - k\omega(t)[1 - \cos\theta(t)] + 2g(t)\sqrt{\bar{M}}\sin\theta(t)\cos[2\omega(t)t + \gamma(t)]\}\hat{S}_z \\ &\quad - \dot{\gamma}(t)[1 - \cos\theta(t)]\hat{S}_z. \end{aligned} \tag{30}$$

The eigenstates of operator $\hat{S}_z$ corresponding to the eigenvalues $S_z = +1$, $S_z = -1$, and $S_z = 0$ are

$$\begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}, \quad \text{and} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix},$$

respectively. The eigenstates of operator $\hat{\bar{M}}$ are

$$\begin{pmatrix}|n\rangle\\0\\0\end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix}0\\|n+1\rangle\\|n+1\rangle\end{pmatrix}, \quad \text{and} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}0\\|n+1\rangle\\-|n+1\rangle\end{pmatrix},$$

respectively. We can obtain three particular solutions of the time-dependent Schrödinger equation (16), respectively. For $S_z = +1$, one has

$$|\psi_{S_z=+1}(t)\rangle = \exp\left\{-i\int_0^t [\dot{\delta}^d_{S_z=+1}(t') + \dot{\delta}^g_{S_z=+1}(t')]dt'\right\}\hat{V}(t)\begin{pmatrix}|n\rangle\\0\\0\end{pmatrix}, \tag{31}$$

where

$$\dot{\delta}^d_{S_z=+1}(t') = n\omega(t') + \omega_0(t')\cos\theta(t') - k\omega(t')[1-\cos\theta(t')] + 2\sqrt{\frac{n!}{(n-k)!}}g(t')\sin\theta(t')\cos[2\omega(t')t' + \gamma(t')], \tag{32}$$

$$\dot{\delta}^g_{S_z=+1}(t') = -\dot{\gamma}(t')[1-\cos\theta(t')], \tag{33}$$

for $S_z = -1$,

$$|\psi_{S_z=-1}(t)\rangle = \exp\left\{-i\int_0^t [\dot{\delta}^d_{S_z=-1}(t') + \dot{\delta}^g_{S_z=-1}(t')]dt'\right\}\hat{V}(t)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\|n+1\rangle\\|n+1\rangle\end{pmatrix}, \tag{34}$$

where

$$\dot{\delta}^d_{S_z=-1}(t') = (n+1)\omega(t') + \omega_0(t')\cos\theta(t') - k\omega(t')[1-\cos\theta(t')] + 2\sqrt{\frac{(n+k+1)!}{(n+1)!}}g(t')\sin\theta(t')\cos[2\omega(t')t' + \gamma(t')], \tag{35}$$

$$\dot{\delta}^g_{S_z=-1}(t') = -\dot{\gamma}(t')[1-\cos\theta(t')], \tag{36}$$

and for $S_z = 0$,

$$|\psi_{S_z=0}(t)\rangle = \exp\left\{-i\int_0^t [\dot{\delta}^d_{S_z=0}(t') + \dot{\delta}^g_{S_z=0}(t')]dt'\right\}\hat{V}(t)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\|n+1\rangle\\-|n+1\rangle\end{pmatrix}, \tag{37}$$

where

$$\dot{\delta}^d_{S_z=0}(t') = (n+1)\omega(t') + \omega_0(t')\cos\theta(t') - k\omega(t')[1-\cos\theta(t')], \tag{38}$$

$$\dot{\delta}^g_{S_z=0}(t') = -\dot{\gamma}(t')[1-\cos\theta(t')]. \tag{39}$$

From (32)–(33), (35)–(36), and (38)–(39), we conclude that the dynamical and the geometric phase factors of the system are $\exp[-i\int_0^t \dot{\delta}^d_{S_z}(t')dt']$ and $\exp[-i\int_0^t \dot{\delta}^g_{S_z}(t')dt']$ with $S_z = \pm 1, 0$, receptively. In particular, when we consider a cycle in the parameter space of the invariant $\hat{I}(t)$ and let $\theta(t) =$ constant, one has from (33), (36) and (39)

$$\delta^g_{S_z}(T) = -2\pi(1-\cos\theta) \quad (S_z = \pm 1, 0), \tag{40}$$

here $2\pi(1-\cos\theta)$ denotes the solid angle over the parameter space of the invariant $\hat{I}(t)$. It is pointed out that the geometric phases in the cycle case have nothing to do with the frequency $\omega(t)$ of the photon field, the coupling coefficient $g(t)$ between photons and atoms, and the atom transition frequency $\omega_0(t)$.

4 Conclusions

In this letter, we have studied the dynamical and the geometric phases in a generalized time-dependent Λ-type k-photon Jaynes-Cummings model with imaginary photon process. We find that the geometric phases in a cycle case are independent of the frequency $\omega(t)$ of the photon field, the coupling coefficient $g(t)$ between photons and atoms, and the atom transition frequency $\omega_0(t)$. If we use the more accuracy device, the geometric phases in this process may be observed, and the geometric phases in this process have the observable physical effect.

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